Flux Integrals

The pictures for problems #1 - #4 are on the last page.

1. Let's orient each of the three pictured surfaces so that the light side is considered to be the "positive" side. Decide whether each of the following flux integrals is positive, negative, or zero. ($\vec{F}$ and $\vec{G}$ are the pictured vector fields.)

   (a) $\iint_{\mathcal{S}_1} \vec{F} \cdot d\vec{S}$.

   (b) $\iint_{\mathcal{S}_2} \vec{F} \cdot d\vec{S}$.

   (c) $\iint_{\mathcal{S}_3} \vec{F} \cdot d\vec{S}$.

   (d) $\iint_{\mathcal{S}_1} \vec{G} \cdot d\vec{S}$.

   (e) $\iint_{\mathcal{S}_2} \vec{G} \cdot d\vec{S}$.

   (f) $\iint_{\mathcal{S}_3} \vec{G} \cdot d\vec{S}$.

2. In each part, you are given an orientation of one of the pictured surfaces. Decide whether this orientation means that the light side or dark side of the surface is the "positive" side, or if the description just doesn't make sense.

   (a) $\mathcal{S}_1$, oriented with normals pointing upward.

   (b) $\mathcal{S}_2$, oriented with normals pointing upward.

   (c) $\mathcal{S}_2$, oriented with normals pointing toward the y-axis.

   (d) $\mathcal{S}_3$, oriented with normals pointing outward.

   (e) $\mathcal{S}_3$, oriented with normals pointing toward the origin.

3. In each part, you are given a parameterization of one of the three pictured surfaces. Decide whether the orientation induced by the parameterization has the light side or dark side of the surface as the "positive" side.

   (a) For $\mathcal{S}_1$, $\vec{r}(u,v) = \langle u, v, \sqrt{u^2+v^2} \rangle$ with $u^2+v^2 < 1$.

(b) For $\mathcal{S}_1$, $\vec{r}(u,v) = \langle u\cos v, u\sin v, u\rangle$ with $0 \le u < 1$ and $0 \le v < 2\pi$.

(c) For $\mathcal{S}_2$, $\vec{r}(u,v) = \langle \cos v, u, \sin v\rangle$ with $-1 < u < 1$ and $0 \le v < 2\pi$.

(d) For $\mathcal{S}_3$, $\vec{r}(u,v) = \langle \sin v\cos u, \sin v\sin u, \cos v\rangle$ with $0 \le u < 2\pi$ and $0 \le v \le \pi$.

4. Compute the following flux integrals (remember that parameterizations of the surfaces are given in #3). Do the signs of your answers agree with your answers to #1?

   (a) $\iint_{\mathcal{S}_1} \vec{F} \cdot d\vec{S}$, where $\mathcal{S}_1$ is oriented with normals pointing upward. ($\vec{F}(x,y,z) = \langle 0,0,-z\rangle$, as before.)

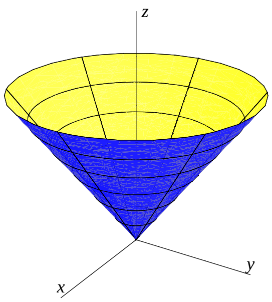
   (b) $\iint_{\mathcal{S}_2} \vec{G} \cdot d\vec{S}$, where $\mathcal{S}_2$ is oriented with normals pointing toward the y-axis. ($\vec{G}(x,y,z) = \langle 0,y,0\rangle$, as before.)

(c) $\iint_{\mathcal{S}_3} \vec{F} \cdot d\vec{S}$, where $\mathcal{S}_3$ is oriented with normals pointing outward. ($\vec{F}(x, y, z) = \langle 0, 0, -z \rangle$, as before.)

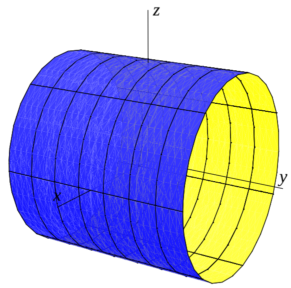
5. Let $\mathcal{S}$ be the portion of the surface $3x - 3y + z = 12$ lying inside the cylinder $x^2 + y^2 = 1$, oriented with normals pointing upward. Let $\vec{F}(x, y, z) = \langle -x^2, 0, -3y^2 \rangle$. Evaluate the flux integral $\iint_{\mathcal{S}} \vec{F} \cdot d\vec{S}$.

These are the surfaces for problems #1 - #4. Each is colored so that one side of the surface is light and the other side is dark.

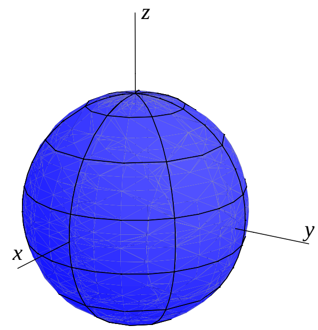
$\mathcal{S}_1$ is the portion of the cone $z = \sqrt{x^2 + y^2}$ under the plane $z = 1$.



$\mathcal{S}_2$ is the portion of the cylinder $x^2 + z^2 = 1$ between the planes $y = -1$ and $y = 1$.

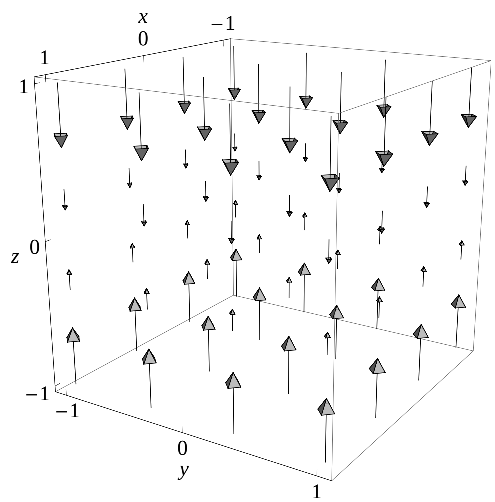


$\mathcal{S}_3$ is the unit sphere $x^2 + y^2 + z^2 = 1$.



These are the vector fields $\vec{F}$ and $\vec{G}$ for problems #1 - #4. (Note that the origin is located in the middle of each box.)

$\vec{F}(x, y, z) = \langle 0, 0, -z \rangle$



$\vec{G}(x, y, z) = \langle 0, y, 0 \rangle$